SECOND TERM SCHEME OF WORK

FURTHER MATHEMATICS

1. **DIFFERENTIATION I: Limit of a function; Differentiation from first principle; differentiation of polynomial functions**
2. **DIFFERENTIATION II: Differentiation of transcendental function such as $sinx, e^{ax}, log3x$**
3. **DIFFERENTIATION III: Rules of differentiation; product rule; quotient rule; function of functions**
4. **DIFFERENTIATION IV: Application of differentiation to**
   a. **Rate of change**
   b. **Gradient**
   c. **Maximum and minimum values**
   d. **Equation of motion**
5. **DIFFERENTIAITON V: Higher derivatives; differentiation of implicit functions.**
6. **BINIOMIAL EXPANSION I: Pascal triangle; Binomial expression of $(a+b)^n$ where n is +ve integer, -ve integer or fractional value**
7. **BINOMIAL EXPANSION II: Finding nth term; application of binomial expansion**
8. **CONIC SECTION(THE CIRCLE) I: Definition of circle; equation of circle given centre and radius**
9. **CONIC SECTION(THE CIRCLE) II: General equation of a circle**
   a. **Finding centre and radius of a given circle**
   b. **Finding equation of a circle given the end point of the diameter**
   c. **Equation of a circle passing through three points**
10. **CONIC SECTION(THE CIRCLE) III: Equation of tangent to a circle; length of tangent to a circle**
11. **Revision**
12. **Examination**
13. **Examination**

THEME:
DATE:
CLASS:
TIME:
PERIOD:
DURATION:
SUBJECT: Further Mathematics

UNIT TOPIC: **Differentiation I**

LESSON TOPIC: **Limit of a function; Differentiation from first principle; differentiation of polynomial functions**

SPECIFIC OBJECTIVES: At the end of the lesson, students should be able to:

1. Find the limit of a function
2. Differentiate from the first principle
3. Differentiate polynomial functions

INSTRUCTIONAL RESOURCES: Charts showing differentiation from first principle

PRESENTATION: (Content Development)

STEP 1: ***Identification of Prior Ideas***

Mode: Group

Teacher's activities: The teacher introduces the topic by asking the students question related to the topic

Students' activities: Students respond to teacher

STEP 2: Exploration

Mode: Individual/group

Teacher's activities: The teacher displays the instructional resources and asks students to identify the resources and states it.

Students' activities: Students carry out teacher's instructions.

STEP 3: Discussion

Mode: Entire class

Teacher's activities: The teacher with the use of instructional resources guides students on the following.

Limit of a Function

The concept of limit of a function is essentially required in the field of calculus. Consider the following functions

$y = x$ $\quad$ $y(x) = x$

$y = 2x^2 + 3x$ $\quad$ $y(x) = 2x^2 + 3x$

$y = 3a^2 + 3a$ $\quad$ $y(a) = 2a^2 + 3a$

Let y be function of x.

If $y = 3x + 1$, then y is a function of x written as $y = f(x)$. The limit at which x tends to 2,

$y = 3(2) + 1$

$y = 6 + 1$

$y = 7$

Differentiation from the First Principle

$If\ y = x$ ------------------------------------ (i)

$y + dy = x + dx$

$dy = x + dx - y$

$from\ equation\ (i), put\ x\ for\ y$

$dy = x + dx - x$

$$\frac{dy}{dx} = \frac{dx}{dx}$$

$$\frac{dy}{dx} = 1$$

Students' activities: Students follow their teacher as they participate in the lesson.

STEP 4: Application
Mode: Group
Teacher's activities: The teacher explains to the students where the concept of Differentiation is applicable
Differentiation is applicable in the field of advanced Mathematics, Computer Studies, Physics and Engineering.
Students' activities: Students follow their teacher.

STEP 5: Evaluation
Mode: Entire class/Individual
Teacher's activities: The teacher evaluates the students by giving them question(s)

1. If $y = 3x^3 + 2x$, solve the equation when the value of x tends to 2
2. Differentiate $y = x^2$ from the first principle
3. Find the derivative of $y = 3x^3$ using the first principle
4. Find the derivative of the following from the first principle
   i. x^2
   ii. $2x + 1$

Students' activities: The students respond to the question(s)

Assignment:
Prove that $\frac{dy}{dx}$ $of\ ax^n$ is equal to nax^{n-1}

References:

New Further Mathematics Project for SSS1

SECOND TERM NOTE
FURTHER MATHEMATICS SSS2
WEEK ONE

DIFFERENTIATION (I)
LIMIT OF A FUNCTION
The concept of limit of a function is essentially required in the field of calculus. Consider the following functions

$y = x$ $\quad\quad$ $y(x) = x$

$y = 2x^2 + 3x$ $\quad\quad$ $y(x) = 2x^2 + 3x$

$y = 3a^2 + 3a$ $\quad\quad$ $y(a) = 2a^2 + 3a$

<u>Example 1</u>

If $y = 3x^2 + 2x$, express the function if the value of x tends to $1, 2, and\ 3$

Solution

When x tends to 1

$\lim_{x \to 1} y = 3x^2 + 2x$

$y = 3(1)^2 + 2(1)$

$y = 3(1) + 2(1)$

$y = 3 + 2$

$y = 5$

When x tends to 2

$\lim_{x \to 1} y = 3x^2 + 2x$

$y = 3(2)^2 + 2(2)$

$y = 3(4) + 2(2)$

$y = 12 + 4$

$y = 16$

When x tends to 3

$\lim_{x \to 1} y = 3x^2 + 2x$

$y = 3(3)^2 + 2(3)$

$y = 3(9) + 2(3)$

$y = 27 + 6$

$y = 33$

Example 1

Solve $y = \frac{3x^2 - 2x}{x}$ for $x = 2$

Solution

$$y = \frac{3x^2 - 2x}{x}$$

$$y = \frac{3(2)^2 - 2(2)}{2}$$

$$y = \frac{3(4) - 2(2)}{2}$$

$$y = \frac{12 - 4}{2}$$

$$y = \frac{16}{2}$$

$y = 8$

DIFFERENTIATION FROM FIRST PRINCIPLE

1. $y = x$
   $y + dy = x + dx$
   $dy = x + dx - y$
   $dy = x + dx - x$
   $dy = dx$
   $\frac{dy}{dx} = \frac{dx}{dx}$
   $\frac{dy}{dx} = 1$

2. $y = x^2$
   $y + dy = (x + dx)^2$
   $y + dy = (x + dx)(x + dx)$
   $y + dy = x^2 + 2xdx + dx^2$
   $dy = x^2 + 2xdx + dx^2 - y$
   $dy = x^2 + 2xdx + dx^2 - x^2$
   $dy = 2xdx + dx^2$
   $\frac{dy}{dx} = \frac{2xdx}{dx} + \frac{dx^2}{dx}$
   $\frac{dy}{dx} = 2x + dx$
   $\lim_{dx \to 0} \frac{dy}{dx} = 2x + 0$
   $\frac{dy}{dx} = 2x$

Hence, generally, the following relation holds

If $y = ax^n$, then, $\frac{dy}{dx} = nax^{n-1}$

DIFFERENTIATION OF POLYNOMIAL FUNCTIONS

<u>Example 3</u>

Differentiate the function $y = 6x^3$

<u>Solution</u>

$y = 6x^3$

$\frac{dy}{dx} = nax^{n-1}$

$\frac{dy}{dx} = 3 \times 6x^{3-1}$

$\frac{dy}{dx} = 18x^2$

<u>Example 4</u>

Differentiate the function $y = \frac{2x^4}{3}$

Solution

$$y = \frac{2x^4}{3}$$

$$\frac{dy}{dx} = nax^{n-1}$$

$$\frac{dy}{dx} = 4 \times \frac{2x^{4-1}}{3}$$

$$\frac{dy}{dx} = \frac{8x^3}{3}$$

THEME:
DATE:
CLASS:
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PERIOD:
DURATION:
SUBJECT: **Further Mathematics**
UNIT TOPIC: **Differentiation II**
LESSON TOPIC: **Differentiation of transcendental function such as** $sinx, e^{ax}, log3x$
SPECIFIC OBJECTIVES: At the end of the lesson, students should be able to:

1. Identify transcendental function
2. Differentiate trigonometric functions
3. Differentiate exponential functions

INSTRUCTIONAL RESOURCES: Charts showing transcendental functions
PRESENTATION: (Content Development)

STEP 1: ***Identification of Prior Ideas***
Mode: Group

Teacher's activities: The teacher introduces the topic by asking the students question related to the topic
Students' activities: Students respond to teacher

STEP 2: Exploration
Mode: Individual/group
Teacher's activities: The teacher displays the instructional resources and asks students to identify the resources and states it.
Students' activities: Students carry out teacher's instructions.

STEP 3: Discussion
Mode: Entire class
Teacher's activities: The teacher with the use of instructional resources guides students on the following.

Transcendental function

Transcendental function includes trigonometric functions, exponential function and logarithmic function.

Differentiation of Trigonometric function

If y is a function of x, such that;

$y = sin\ x$, then $\frac{dy}{dx} = cosx$

$y = cosx$, then $\frac{dy}{dx} = -sinx$

$y = tanx$, then $\frac{dy}{dx} = sec^2x$

Differentiation of Exponential function

If y is a function of x, such that;

$y = e^x$, then $\frac{dy}{dx} = e^x$

$y = e^{ax}$, then $\frac{dy}{dx} = ae^{ax}$

Students' activities: Students follow their teacher as they participate in the lesson.

STEP 4: Application
Mode: Group
Teacher's activities: The teacher explains to the students where the concept of Differentiation is applicable
Differentiation is applicable in the field of advanced Mathematics, Computer Studies, Physics and Engineering.
Students' activities: Students follow their teacher.

STEP 5: Evaluation
Mode: Entire class/Individual
Teacher's activities: The teacher evaluates the students by giving them question(s)

1. Give two examples each of logarithmic and exponential function
2. Differentiate $y = e^x$
3. Find the derivative of $y = 3e^{3x}$
4. Find the derivative of the following from the first principle
   i. $y = sinx + e^x$
   ii. $y = sinx - cosx$
   iii. $y = cosx + 2e^{2x}$

Students' activities: The students respond to the question(s)

Assignment:
Find the derivative of $y = sin3x - 3e^{-2x}$
References:

New Further Mathematics Project for SSS1

SECOND TERM NOTE

FURTHER MATHEMATICS SSS2
WEEK TWO

DIFFERENTIATION (II)
DIFFERENTIATION OF TRIGONOMETRIC FUNCTIONS

If $y = sin\ x, \frac{dy}{dx} = cosx$

If $y = cosx, \frac{dy}{dx} = -sinx$

If $y = tanx, \frac{dy}{dx} = sec^2x$

<u>Example 1</u>

Find the $\frac{dy}{dx}$ of the following functions

i. $y = x^2 + cosx$
ii. $y = x^3cosx$
iii. $y = \frac{sinx}{cosx}$

<u>Solution</u>

i. $y = x^2 + cosx$

$$\frac{dy}{dx} = \frac{d}{dx}x^2 + \frac{d}{dx}cosx$$
$$\frac{dy}{dx} = 2 \times x^{2-1} + (-sinx)$$
$$\frac{dy}{dx} = 2x^1 + (-sinx)$$
$$\frac{dy}{dx} = 2x - sinx$$

iv. $y = x^3 cosx$

Let $U = x^3$ and $V = cosx$

$\frac{dU}{dx} = 3x^2$ and $\frac{dV}{dx} = -sinx$

$$\frac{dy}{dx} = U\frac{dV}{dx} + V\frac{dU}{dx}$$
$$\frac{dy}{dx} = x^3(-sin) + cosx(3x^2)$$
$$\frac{dy}{dx} = -x^3 sin + 3x^2 cosx$$

THEME:
DATE:
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PERIOD:
DURATION:
SUBJECT: **Further Mathematics**
UNIT TOPIC: **Differentiation III**
LESSON TOPIC: **Differentiation Rules (Sum, Difference, Product and Quotient rules)**
SPECIFIC OBJECTIVES: At the end of the lesson, students should be able to:

1. Identify the rules of Differentiation
2. Apply sum and difference rules
3. Apply product and quotient rules

INSTRUCTIONAL RESOURCES: Charts showing rules of Differentiations
PRESENTATION: (Content Development)

STEP 1: ***Identification of Prior Ideas***
Mode: Group
Teacher's activities: The teacher introduces the topic by asking the students question related to the topic
Students' activities: Students respond to teacher

STEP 2: Exploration
Mode: Individual/group

Teacher's activities: The teacher displays the instructional resources and asks students to identify the resources and states it.
Students' activities: Students carry out teacher's instructions.

STEP 3: Discussion
Mode: Entire class
Teacher's activities: The teacher with the use of instructional resources guides students on the following.
Rules of Differentiaiton

- **Sum and Difference rules:** if y, U and V are functions of x such that
  $y = U + V \ or \ y = U - V$ then,
  $\frac{dy}{dx} = \frac{dU}{dx} + \frac{dV}{dx}$ -----------------------------------(sum rule)
  And
  $\frac{dy}{dx} = \frac{dU}{dx} - \frac{dV}{dx}$ -----------------------------------(difference rule)
- **Product and Quotient rules:** if y, U and V are functions of x such that
  $y = UV \ or \ y = \frac{U}{V}$then,
  $\frac{dy}{dx} = U\frac{dV}{dx} + V\frac{dU}{dx}$ ------------------------------------(Prouduct rule)
  $\frac{dy}{dx} = \frac{V\frac{dU}{dx} - U\frac{dV}{dx}}{V^2}$ ------------------------------------(Quotient rule)

Students' activities: Students follow their teacher as they participate in the lesson.

STEP 4: Application
Mode: Group
Teacher's activities: The teacher explains to the students where the concept of Differentiation is applicable
Differentiation is applicable in the field of advanced Mathematics, Computer Studies, Physics and Engineering.
Students' activities: Students follow their teacher.

STEP 5: Evaluation
Mode: Entire class/Individual
Teacher's activities: The teacher evaluates the students by giving them question(s)
1. State Sum and Difference Rules
2. State Product and Quotient Rules
3. Find the derivative of $y = 3x^3 + x^2 - 5x$
4. Find the derivative of the following
   i. $\frac{2x^3}{x^2}$
   ii. $(2x + 1)(2x - 1)$

Students' activities: The students respond to the question(s)

Assignment:
1. Prove the sum rule
2. Find the $\frac{dy}{dx}$ of the function $\frac{3x^3(2x-3)}{x^2}$

References:

SECOND TERM NOTE

FURTHER MATHEMATICS SSS2

WEEK THREE

DIFFERENTIATION (III)

RULES OF DIFFERENTIATION

SUM RULE AND DIFFERENCE RULE

If y, U and V are functions of x such that $y(x) = U(x) + V(x)$ or $y(x) = U(x) - V(x)$ then,

$\frac{dy}{dx} = \frac{dU}{dx} + \frac{dV}{dx}$ -----------------------------sum rule

$\frac{dy}{dx} = \frac{dU}{dx} - \frac{dV}{dx}$ -----------------------------difference rule

<u>Example1</u>

Find the derivative of the following functions

i. $y = 2x^2 - 5x + 2$
ii. $y = 3x^2 + \frac{1}{x}$
iii. $y = \frac{2}{\sqrt{x}} + \sqrt{x}$

<u>Solution</u>

i. $y = 2x^2 - 5x + 2$

$$\frac{dy}{dx} = \frac{d}{dx}2x^2 - \frac{d}{dx}5x + \frac{d}{dx}2$$

$$\frac{dy}{dx} = 2 \times 2x^{2-1} - 1 \times 5x^{1-1} + 0$$

$$\frac{dy}{dx} = 4x^1 - 5x^0$$

$$\frac{dy}{dx} = 4x - 5$$

ii. $y = 3x^2 + \frac{1}{x}$

$$\frac{dy}{dx} = \frac{d}{dx}3x^2 + \frac{d}{dx}\frac{1}{x}$$
$$\frac{dy}{dx} = \frac{d}{dx}3x^2 + \frac{d}{dx}x^{-1}$$
$$\frac{dy}{dx} = 2 \times 3x^{2-1} + (-1)x^{-1-1}$$
$$\frac{dy}{dx} = 6x^1 + (-1)x^{-2}$$
$$\frac{dy}{dx} = 6x - x^{-2}$$

PRODUCT RULE AND QUOTIENT RULE

If y, U and V are functions of x such that $y(x) = U(x).V(x)$ or $y(x) = \frac{U(x)}{V(x)}$then,

$\frac{dy}{dx} = U\frac{dV}{dx} + V\frac{dU}{dx}$ ------------------------------product rule

$\frac{dy}{dx} = \frac{V\frac{dU}{dx} - U\frac{dV}{dx}}{V^2}$ ------------------------------quotient rule

<u>Example2</u>

Find the $\frac{dy}{dx}$ of the following functions

i. $y = 2x^3.x^2$
ii. $y = (3x - 2x^2)(5 + 4x)$
iii. $y = \frac{2x^3}{x^2}$
iv. $y = \frac{x^3+2x}{x-1}$

<u>Solution</u>

i. $y = 2x^3.x^2$

Let $U = 2x^3\ and\ V = x^2$

$\frac{dU}{dx} = 6x^2$ and $\frac{dV}{dx} = 2x$

$$\frac{dy}{dx} = U\frac{dV}{dx} + V\frac{dU}{dx}$$
$$\frac{dy}{dx} = 2x^3\ .2x + x^2 6x^2$$
$$\frac{dy}{dx} = 4x^4 + 6x^4$$
$$\frac{dy}{dx} = 10x^4$$

FUNCTION OF A FUNCTION

Suppose that we know y is a function of U and U itself is also a function of x, then, to find $\frac{dy}{dx}$, the following relation holds

$\frac{dy}{dx} = \frac{dy}{dU} \times \frac{dU}{dx}$

The relation above is called the Chain's rule

Example 3

Find the derivative of the following

i. $y = (x^2)^3$
ii. $y = (3x^2 - 2)^3$
iii. $y = \sqrt{(1 - 2x^2)}$

Solution

i. $y = (x^2)^3$

Let $U = x^2$; $\frac{dy}{dx} = 2x$

Then, $y = U^3$ $\frac{dy}{dx} = 3U^2$

$\frac{dy}{dx} = \frac{dy}{dU} \times \frac{dU}{dx}$

$\frac{dy}{dx} = 3U^2 . 2x$

$\frac{dy}{dx} = 6xU^2$

$$\frac{dy}{dx} = 6x. (x^2)^2$$

$$\frac{dy}{dx} = 6x. x^4$$

$$\frac{dy}{dx} = 6x^5$$

THEME:
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SUBJECT: **Further Mathematics**
UNIT TOPIC: **Differentiation IV**
LESSON TOPIC: ***Application of differentiation to rate of change and equation of motion***

SPECIFIC OBJECTIVES: At the end of the lesson, students should be able to:

1. Apply differentiation to rate of change of velocity
2. Apply differentiation to rate of change of motion
3. Apply the rules of differentiation in rate of change quantities

INSTRUCTIONAL RESOURCES: Charts showing applications of differentiation
PRESENTATION: (Content Development)

STEP 1: ***Identification of Prior Ideas***
Mode: Group
Teacher's activities: The teacher introduces the topic by asking the students question related to the topic
Students' activities: Students respond to teacher

STEP 2: Exploration
Mode: Individual/group
Teacher's activities: The teacher displays the instructional resources and asks students to identify the resources and states it.
Students' activities: Students carry out teacher's instructions.

STEP 3: Discussion
Mode: Entire class
Teacher's activities: The teacher with the use of instructional resources guides students on the following.
RATE OF CHANGE

If y is a function of x, $\frac{dy}{dx}$ can sometimes be interpreted as the rate at which y is changing with respect to x. If y increases as x increases, $\frac{dy}{dx} > 0$, while If y decreases as x decreases, $\frac{dy}{dx} < 0$.
Students' activities: Students follow their teacher as they participate in the lesson.

STEP 4: Application
Mode: Group
Teacher's activities: The teacher explains to the students where the concept of Differentiation is applicable
Differentiation is applicable in the field of advanced Mathematics, Computer Studies, Physics and Engineering.
Students' activities: Students follow their teacher.

STEP 5: Evaluation
Mode: Entire class/Individual
Teacher's activities: The teacher evaluates the students by giving them question(s)

1. If x is the rate of change of p to q, write an equation to illustrate the information
2. The radius of a circle is increasing at the rate of 0.5cm/s. Find the rate at which the area is increasing when the radius of the circle is 7cm.
3. The motion of a particle starting from 0, is described by the equation, $S = ut + \frac{1}{2}t^2$. How far is the particle from 0, when the particle is momentarily at rest?

Students' activities: The students respond to the question(s)

Assignment:
If the acceleration of moving particle is given by $a = t^5 + \frac{5}{2}t^3 + t$, find the velocity

References:

New Further Mathematics Project for SSS1

SECOND TERM NOTE
FURTHER MATHEMATICS SSS2
WEEK FOUR

DIFFERENTIATION (IV)

Application of differentiation to rate of change and equation of motion

RATE OF CHANGE

If y is a function of x, $\frac{dy}{dx}$ can sometimes be interpreted as the rate at which y is changing with respect to x. If y increases as x increases, $\frac{dy}{dx} > 0$, while If y decreases as x decreases, $\frac{dy}{dx} < 0$.

Example 1

The radius of a circle is increasing at the rate of 0.01cm/s. Find the rate at which the area is increasing when the radius of the circle is 5cm.

Solution

Let A be the area of circle of radius r.

$$\frac{dr}{dt} = 0.01cm/s$$

$$A = \pi r^2$$

$$\frac{dA}{dr} = 2\pi r$$

By the chain rule,

$$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi \times 5 \times 0.01$$

$$\frac{dA}{dt} = 10\pi \times 0.01$$

$$\frac{dA}{dt} = 0.1\pi = 0.1 \times 3.142$$

$$\frac{dA}{dt} = 0.3142cm^2/s$$

Example 2

The motion of a particle starting from 0, is described by the equation, $x = \frac{t^3}{3} - \frac{7}{2}t^2 + 10t$. How far is the particle from 0, when the particle is momentarily at rest?

Example 3

The motion of a particle from O, is described by the equation $S = \frac{2}{3}t^3 - \frac{17}{2}t^2 + 21t$ where S is the distance in metres, and t the time in seconds. Find the acceleration of the particle when it is momentarily at rest.

THEME:
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DURATION:
SUBJECT: **Further Mathematics**
UNIT TOPIC: **Differentiation V**
LESSON TOPIC: ***Higher Derivatives***

SPECIFIC OBJECTIVES: At the end of the lesson, students should be able to:

1. Identify higher derivatives
2. Resolve first, second and third derivatives of functions
3. Solve nth derivative of functions

INSTRUCTIONAL RESOURCES: Charts showing higher derivatives (nth derivatives)
PRESENTATION: (Content Development)

STEP 1: ***Identification of Prior Ideas***
Mode: Group
Teacher's activities: The teacher introduces the topic by asking the students question related to the topic
Students' activities: Students respond to teacher

STEP 2: Exploration
Mode: Individual/group
Teacher's activities: The teacher displays the instructional resources and asks students to identify the resources and states it.
Students' activities: Students carry out teacher's instructions.

STEP 3: Discussion
Mode: Entire class
Teacher's activities: The teacher with the use of instructional resources guides students on the following.
Higher Derivatives
Given that y is a function of x, $\frac{dy}{dx}$ is also a function of x. The derivative $\frac{dy}{dx}$ with respect to x is $\frac{d}{dx}\left(\frac{dy}{dx}\right)$.

$\frac{d}{dx}(\frac{dy}{dx})$ is called the second derivative of y with respect to x, and it is usually denoted as $\frac{d^2y}{dx^2}$ (read, dee two y dee x squared).

Since $\frac{d^2y}{dx^2}$ is also a function of x, successive derivatives can be found.

The third derivative of y with respect to x is $\frac{d^3y}{dx^3}$.

Similarly, $\frac{d^4y}{dx^4}$ is the fourth derivative of y with respect to x.

Students' activities: Students follow their teacher as they participate in the lesson.

STEP 4: Application
Mode: Group
Teacher's activities: The teacher explains to the students where the concept of Differentiation is applicable

Differentiation is applicable in the field of advanced Mathematics, Computer Studies, Physics and Engineering.
Students' activities: Students follow their teacher.

STEP 5: Evaluation
Mode: Entire class/Individual
Teacher's activities: The teacher evaluates the students by giving them question(s)

1. How do you pronounce , $\frac{d^5y}{dx^5}$
2. Find the first, second and third derivatives of $2x^6$
3. Find the first, second and third derivatives of each of the following
   i. $sinx$
   ii. $cosx$

Students' activities: The students respond to the question(s)

Assignment:
Find the first, second and third derivatives of each of the following:

a. $3x^3 - x^4$
b. $-sinx$
c. $cosx + sinx$

References:

New Further Mathematics Project for SSS1

SECOND TERM NOTE
FURTHER MATHEMATICS SSS2
WEEK FIVE

DIFFERENTIATION (V)
Higher Derivatives
Given that y is a function of x, $\frac{dy}{dx}$ is also a function of x. The derivative $\frac{dy}{dx}$ with respect to x is $\frac{d}{dx}\left(\frac{dy}{dx}\right)$.

$\frac{d}{dx}(\frac{dy}{dx})$ is called the second derivative of y with respect to x, and it is usually denoted as $\frac{d^2y}{dx^2}$ (read, dee two y dee x squared).

Since $\frac{d^2y}{dx^2}$ is also a function of x, successive derivatives can be found.

The third derivative of y with respect to x is $\frac{d^3y}{dx^3}$.

Similarly, $\frac{d^4y}{dx^4}$ is the fourth derivative of y with respect to x.

<u>Example 1</u>

Find the first, second and third derivatives of $3x^4$

<u>Solution</u>

Let $y = 3x^4$

Then $\frac{d}{dx} = 12x^3$

$$\frac{d^2y}{dx^2} = 36x^2$$

$$\frac{d^3y}{dx^3} = 72x$$

<u>Example 2</u>

Find the first, second and third derivatives of each of the following:

d. $3x^5 - 2x^4 + x^2 - 1$
e. $sinx$
f. $cosx$

<u>Example 3</u>

Find the first and second derivative of sin^3x

THEME:
DATE:
CLASS:
TIME:
PERIOD:
DURATION:
SUBJECT: **Further Mathematics**
UNIT TOPIC: **Binomial Expression**
LESSON TOPIC: ***Pascal triangle; Binomial expression of (a+b)^n where n is +ve integer, -ve integer or fractional value***

SPECIFIC OBJECTIVES: At the end of the lesson, students should be able to:

1. Formulate Pascal's Triangle
2. Use Pascal's triangle to expand expression
3. Use formula to resolve binomial expression

INSTRUCTIONAL RESOURCES: Charts showing Pascal's Triangle
PRESENTATION: (Content Development)

STEP 1: ***Identification of Prior Ideas***
Mode: Group
Teacher's activities: The teacher introduces the topic by asking the students question related to the topic
Students' activities: Students respond to teacher

STEP 2: Exploration
Mode: Individual/group
Teacher's activities: The teacher displays the instructional resources and asks students to identify the resources and states it.
Students' activities: Students carry out teacher's instructions.

STEP 3: Discussion
Mode: Entire class
Teacher's activities: The teacher with the use of instructional resources guides students on the following.

Pascal's Triangle

Binomial Expansion of $(x+y)^n$ where n is +ve integer, -ve integer or fraction

Consider the expansions of each of the following:

$(x+y)^0, (x+y)^1, (x+y)^2, (x+y)^3, (x+y)^4$

$(x+y)^0 = 1$

$(x+y)^1 = 1x + 1y$

$(x+y)^2 = 1x^2 + 2xy + 1y^2$

$(x+y)^3 = 1x^3 + 3x^2y + 3xy^2 + y^2$

The coefficients of x and y can be displayed in an array as:

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

The array of coefficients displayed above is called the Pascal's triangle, and it is used in determining the co-efficient of the terms of the powers of a binomial expression.

Two significant features of a Pascal's triangle are:

(a) Each line or coefficients is symmetrical
(b) Each line of coefficients can be obtained from the line of coefficients immediately preceding it.

Students' activities: Students follow their teacher as they participate in the lesson.

STEP 4: Application
Mode: Group
Teacher's activities: The teacher explains to the students where the concept of Differentiation is applicable
Pascal's Triangle is applicable in the expansion of bracket with complex or multiple power
Students' activities: Students follow their teacher.

STEP 5: Evaluation
Mode: Entire class/Individual
Teacher's activities: The teacher evaluates the students by giving them question(s)

1. Construct Pascal's triangle whose power of unknown variable is 10
2. Use the Pascal's triangle to express $(x+y)^3$
3. Using Pascal's triangle, expand and simplify completely $(1+2x)^5$

Students' activities: The students respond to the question(s)

Assignment:
Using Pascal's triangle, expand and simplify completely $(x-2y)^5$

References:

New Further Mathematics Project for SSS1

SECOND TERM NOTE
FURTHER MATHEMATICS SSS2
WEEK SIX

BINOMIAL EXPANSION

Pascal's Triangle

Binomial Expansion of $(x+y)^n$ where n is +ve integer, -ve integer or fraction

Consider the expansions of each of the following:

$(x+y)^0, (x+y)^1, (x+y)^2, (x+y)^3, (x+y)^4$

$(x+y)^0 = 1$

$(x+y)^1 = 1x + 1y$

$(x+y)^2 = 1x^2 + 2xy + 1y^2$

$(x+y)^3 = 1x^3 + 3x^2y + 3xy^2 + y^2$

$(x+y)^4 = 1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$

The coefficients of x and y can be displayed in an array as:

$$
\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}
$$

The array of coefficients displayed above is called the Pascal's triangle, and it is used in determining the co-efficient of the terms of the powers of a binomial expression.

Two significant features of a Pascal's triangle are:

(c) Each line or coefficients is symmetrical
(d) Each line of coefficients can be obtained from the line of coefficients immediately preceding it.

Example 1

Using Pascal's triangle, expand and simplify the following completely:

i. $(x+y)^3$
ii. $(2x+3y)^4$

Solution

i. $(x+y)^3$
$= 1x^3y^0 + 3x^2y^1 + 3x^1y^2 + 1x^0y^3$
$= x^3 + 3x^2y + 3xy^2 + y^3$

ii. $(2x+3y)^4$
$= 1(2x)^4(3y)^0 + 4(2x)^3(3y)^1 + 6(2x)^2(3y)^2 + 4(2x)^1(3y)^3 + 1(2x)^0(3y)^4$
$= 16x^4 + 96x^3y + 216x^2y^2 + 216xy^3 + 81y^4$

Example 3

Using Pascal's triangle, expand and simplify completely $(1+2x)^5$

Example 4

Using Pascal's triangle, expand and simplify completely $(x-2y)^5$

THEME:
DATE:
CLASS:
TIME:
PERIOD:
DURATION:
SUBJECT: **Further Mathematics**
UNIT TOPIC: **Binomial Expression**
LESSON TOPIC: **The Binomial Expansion Formula; *Application of Binomial Expansion***

SPECIFIC OBJECTIVES: At the end of the lesson, students should be able to:

1. Identify Binomial expansion formula
2. Use Binomial expansion formula
3. Apply Binomial expansion to solve question
4. Solve more related question

INSTRUCTIONAL RESOURCES: Charts showing Binomial Expansion formula
PRESENTATION: (Content Development)

STEP 1: ***Identification of Prior Ideas***
Mode: Group
Teacher's activities: The teacher introduces the topic by asking the students question related to the topic
Students' activities: Students respond to teacher

STEP 2: Exploration
Mode: Individual/group
Teacher's activities: The teacher displays the instructional resources and asks students to identify the resources and states it.
Students' activities: Students carry out teacher's instructions.

STEP 3: Discussion
Mode: Entire class
Teacher's activities: The teacher with the use of instructional resources guides students on the following.

The Binomial Expansion Formula

Consider the expansion of $(x+y)^5$again:

$(x+y)^5 = (x+y)(x+y)(x+y)(x+y)(x+y)$

The first term is obtained by multiplying the xs in the five brackets. There is only one way of doin this.

The second term is obtained by multiplying any y in one bracket by the xs in the remaining 4 brackets. The number of ways of doing this is 5_{C_1}

Consider the expansions of each of the following:

$(x+y)^0, (x+y)^1, (x+y)^2, (x+y)^3, (x+y)^4$

$(x+y)^0 = 1$

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$(x+y)^2 = 1x^2 + 2xy + 1y^2$

$(x+y)^3 = 1x^3 + 3x^2y + 3xy^2 + y^2$

The coefficients of x and y can be displayed in an array as:

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1 1

1 2 1

1 3 3 1

1 4 6 4 1

The array of coefficients displayed above is called the Pascal's triangle, and it is used in determining the co-efficient of the terms of the powers of a binomial expression.

Two significant features of a Pascal's triangle are:

(e) Each line or coefficients is symmetrical
(f) Each line of coefficients can be obtained from the line of coefficients immediately preceding it.

Students' activities: Students follow their teacher as they participate in the lesson.

STEP 4: Application
Mode: Group
Teacher's activities: The teacher explains to the students where the concept of Differentiation is applicable
Pascal's Triangle is applicable in the expansion of bracket with complex or multiple power
Students' activities: Students follow their teacher.

STEP 5: Evaluation
Mode: Entire class/Individual
Teacher's activities: The teacher evaluates the students by giving them question(s)

4. Construct Pascal's triangle whose power of unknown variable is 10
5. Use the Pascal's triangle to express $(x+y)^3$
6. Using Pascal's triangle, expand and simplify completely $(1+2x)^5$

Students' activities: The students respond to the question(s)

Assignment:
Using Pascal's triangle, expand and simplify completely $(x-2y)^5$

References:

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SECOND TERM NOTE
FURTHER MATHEMATICS SSS2
WEEK SIX

BINOMIAL EXPANSION

Pascal's Triangle

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$(x+y)^4 = 1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$

The coefficients of x and y can be displayed in an array as:

1

1 1

			1		2		1			
		1		3		3		1		
	1		4		6		4		1	
1		5		10		10		5		1

The array of coefficients displayed above is called the Pascal's triangle, and it is used in determining the co-efficient of the terms of the powers of a binomial expression.

Two significant features of a Pascal's triangle are:

(g) Each line or coefficients is symmetrical
(h) Each line of coefficients can be obtained from the line of coefficients immediately preceding it.

Example 1

Using Pascal's triangle, expand and simplify the following completely:

iii. $(x+y)^3$
iv. $(2x+3y)^4$

Solution

iii. $(x+y)^3$

$= 1x^3y^0 + 3x^2y^1 + 3x^1y^2 + 1x^0y^3$

$= x^3 + 3x^2y + 3xy^2 + y^3$

iv. $(2x+3y)^4$

$= 1(2x)^4(3y)^0 + 4(2x)^3(3y)^1 + 6(2x)^2(3y)^2 + 4(2x)^1(3y)^3 + 1(2x)^0(3y)^4$

$= 16x^4 + 96x^3y + 216x^2y^2 + 216xy^3 + 81y^4$

Example 3

Using Pascal's triangle, expand and simplify completely $(1+2x)^5$

Example 4

Using Pascal's triangle, expand and simplify completely $(x-2y)^5$